Biological Information Processing Requires Quantum Logic

Tetsu Oi

Central Research Laboratory, Hitachi, Ltd., Kokubunji, Tokyo 185, Japan

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Chaos dynamics, which characterizes biological information processing, generates information along the course of temporal development of the relevant system. In this system, the macroscopic uncertainty principle holds between observation time Δt and phase space volume $\Delta\Omega$ determined by this observation. In other words Δt and $\Delta\Omega$ cannot simultaneously be small. This principle corresponds to the microscopic uncertainty principle that holds in quantum physics. Through an analogy to this correspondence, it is shown that quantum logic might also govern such macroscopic phenomena as are governed by chaos dynamics.

Introduction

It has recently been proposed by Harth [1], Guevara *et al.* [2], Nicolis [3, 4], Tsuda [5], Tsuda *et al.* [6], and Oi [7], that such important features of biological information processing as memory and cognition are governed by chaos dynamics. In this proposition, first, biological information processing is characterized by information production rather than tautological information transformation. Second, chaos dynamics is taken into account because it seems to be the best physical model of information production.

Another characteristic of chaos dynamics, however, is that overall behavior is unpredictable. Actually this notion is equivalent to information production. Therefore, it is worthwhile to explain the origin of this unpredictability, as indicated below.

The behavior of a trajectory of chaos dynamics does obey a deterministic law. Long-term behavior of the trajectory, however, is stochastic due to the instability inherent in the dynamics. It naturally results in unpredictability. Signals generated by chaos dynamics, therefore, are irregular and random. These signals often fail to be recognized as signals, and are overlooked being judged as mere noise.

Even when it is evident that they are definitely signals, each pulse cannot have any fixed meaning because of its randomness. Instead, a certain statistical value at most representing a sequence of pulse signals is attributed to meaning, as shown by Musha [8]. This feature is what von Neumann [9] observed some thirty years ago as the mode of information transmission by pulse sequences in a nerve cell. He conjectured, therefore, that such a logical operation that required an assignment of a Boolean value on a separate signal pulse could not be executed in living matter, but that some other logic must exist.

The most recent knowledge in neurophysiology and theoretical physics tells us that what he observed at that time actually is a signal sequence in which the power spectrum of phase turbulence is proportional to the inverse of the frequency, *f*. Signals with this feature usually are termed $1/f$ noise.

As has been explained, it has become evident that a signal sequence with a stochastic nature plays an essential role in biological information processing, and that the mechanism generating such a signal sequence is chaos dynamics. These understandings remain, however, within the scope of physics. In order to build a guiding principle for the design of a computer which simulates biological information processing, the understandings mentioned above should be described in terms of the fundamental law of novel logical operation. The author, to the best of his knowledge, does not know of any work relating to this subject except for the preliminary work on inductive inference by the author himself [7].

In this report therefore, a theory directed at finding a clue to build non-Boolean logic governing biological information processing obeying chaos dynamics is presented. The essential point of the theory is that quantum logic which is reviewed by Takeuchi [10] is taken into consideration as non-Boolean logic suitable to describe macroscopic uncertainty due to chaos dynamics. It had previously been considered only as logic describing microscopic uncertainty. In

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the theory presented in this paper the concept of macroscopic uncertainty is due to Tsuda [5] and Prigogine [11]. Moreover, compatibility of the present theory with fundamental laws of biological information processing due to Tomita [12, 13], is discussed.

Macroscopic uncertainty principle

Let us observe how chaos dynamics generates macroscopic uncertainty, according to the interpretation by Tsuda [5]. To represent chaos, a one-dimensional map expressed as Eqn. (1) is considered.

$$f(x_n) = \lambda x_n(1-x_n) \quad (1)$$

where $n \in N$, $x_n \in R$, $0 < x_0 < 1$, $\lambda > 3.85$.

A trajectory starting from a certain initial value x_0 moves randomly in an area (0, 1). Assigning a symbol "0" to a status when the trajectory resides in (0, 1/2) and symbol "1" to (1/2, 1) allows for the trajectory to be represented in terms of dyadic expansion, to be 0.0100101110..., for instance.

In such a representation, the greater the order of expansion, the finer the observation of the relevant state of the dynamics. In other words, the right side of the number sequence represents the microscopic state, whereas the left side represents the macroscopic state. Rounding off of the sequence at a finite length corresponds to soaking the system in a heat bath with a temperature equivalent to the lowest limit of the relevant finite length of the number sequence.

In the case of a chaotic dynamical system, only an infinite dyadic expansion can uniquely define an initial state of the system. Actually, however, observation time must be limited to a finite length, however long that is. Therefore, uncertainty is inevitably introduced into the initial state of the system.

In other words, for the system obeying chaos dynamics, one cannot simultaneously retain short observation time Δt and small volume of state space $\Delta\Omega$ which can be determined by relevant observation. Here, the logarithm of the inverse of $\Delta\Omega$ is information gain, ΔI, or $\Delta I = -\log_2 \Delta\Omega$, $\Delta\Omega = 2^{-\Delta I}$. This statement will be represented as Eqn. (2), where K is constant, although it is not known if any universal constant exists or not. This law can be termed the macroscopic uncertainty principle, analogous to the uncertainty principle in quantum physics.

$$\Delta t \cdot \Delta\Omega = 2^{-\Delta I} \cdot \Delta t \geqq K. \quad (2)$$

Moreover, Prigogine [11] has recently examined in detail the physical meaning of time and entropy (or, information) appearing in an unstable dynamical system such as chaos. According to his discussion, time in an unstable dynamical system is no more a mere external variable, but must be considered as an operator (time operator) expressing the internal state of the relevant system. This treatment requires that entropy also be considered as an entropy operator. Moreover, these two operators have a mutually unexchangeable relationship. This relationship is parallel to the unexchangeable relationship between operators representing position and momentum. Therefore, unexchangeability between the time operator and the entropy operator in a chaos system also results in the macroscopic uncertainty principle.

Quantum logic

Suppose one is to check if a statement represented by a chaotic phenomenon is true or not. For instance, one is to observe a turbulence pattern in a fluid and to respond to the statement that "deviation of the pattern is within $\Delta\Gamma$ from a particular pattern", either affirmatively or negatively. The result may be either [1] true, [2] false, or, according to the macroscopic uncertainty in chaos dynamics, [3] not specified. These results need to be represented by many-value logic other than the classical two-valued logic. Such nonclassical logic having three or more truth values as modal logic, intuitionistic logic and quantum logic have been devised, as has been summarized by Sugihara [14]. Among them quantum logic has been constructed based on the microscopic uncertainty principle dominating the world of quantum physics. Therefore, this logic is the easiest to be considered as having a parallel relationship to the macroscopic uncertainty principle discussed in this paper. Therefore, the outline of the quantum logic will be reviewed, and the significance of this application to such a macroscopic system as chaos will also be discussed.

As is well known, such an uncertainty principle as stating that the position and momentum of a particle cannot simultaneously be measured exactly dominates the world of quantum physics. In this world the following observational proposition will be taken into consideration: Observation of a certain physical quantity results in obtaining a certain value. In quantum physics, observation of either position or momentum is a mutually incompatible proposition, contrary to their mutual compatibility in the world of classical dynamical systems.

Let us denote these two propositions by symbols A and B. If these two propositions were compatible, such a proposition formula as follows would hold:

$$A \Leftrightarrow (A \wedge B) \vee (A \wedge \neg B), \quad (3)$$

where symbols $\Leftrightarrow$, $\wedge$, $\vee$, and $\neg$ represent "equivalence", "and", "or", and "negation", respectively. This formula represents that the truth of proposition A is assured independent of the truth value of proposition B. In the case that A is compatible to B, B is also compatible to A, leading to the following formula to hold simultaneously:

$$B \Leftrightarrow (A \wedge B) \vee (\neg A \wedge B). \quad (4)$$

Actually, however, these two are not compatible. Formulae (3) and (4), therefore, do not hold. For instance, the right side value of formula (3) is not A. On the other hand, the following formula holds:

$$A \wedge (B \vee \neg B) \Leftrightarrow A.$$

Therefore, the following formula does not hold when A and B are mutually incompatible.

$$A \wedge (B \vee \neg B) \Leftrightarrow (A \wedge B) \vee (A \wedge \neg B). \quad (5)$$

Generalizing this formula by substituting C for $\neg$ B, the law of partition as formula (6) is derived. In other words, when either A and B or A and C are mutually incompatible, the following partition law does not hold:

$$A \wedge (B \vee C) \Leftrightarrow (A \wedge B) \vee (A \wedge C). \quad (6)$$

Equivalently, the other partition law does not hold either:

$$A \vee (B \wedge C) \Leftrightarrow (A \vee B) \wedge (A \vee C). \quad (7)$$

As shown above, the characteristic of quantum logic lies in the statement that a partition law does not hold as a consequence of uncertainty or, in other words, incompatibility of propositions. This characteristic can also be represented as logic in a linear partial space where quantum physics holds.

So far, however, this quantum logic has not been recognized as being very useful. The reason might be that the world dominated by the uncertainty principle has heretofore been believed to be limited to a microscopic area where quantum physics dominates. This practical world might, however, also be governed by quantum logic where there are truth values YES, NO, and NOT SPECIFIED, as pointed out by Takeuchi [10]. Moreover, the macroscopic uncertainty principle does hold in the chaos dynamics system which describes a macroscopic world, as pointed out by Tsuda. Therefore, it should be of great significance to apply quantum logic instead of classical logic to such a macroscopic system governed by chaos dynamics as description and organization of semantic information as well as cognition.

The arguments stated above relating to the route of transforming the knowledge of chaos physics to logical rule in computer science may be summarized as follows:

⟨1⟩ The stochastic nature of chaos dynamics results in uncertainty between observation time and the exponent of the inverse of information gain;

⟨2⟩ this uncertainty leads to a proposition stating that an observational proposition of "determining observation time" and another observational proposition of "determining information gain" are incompatible;

⟨3⟩ this incompatibility is described by a logical law characterized by failure of the law of partition between these observational propositions. Quantum logic is a typical example of the logical law of this nature.

Although this logic remains to be utilized in the future, arguments concerning the possible applicability of quantum logic to pattern recognition is inspiring. Watanabe [15] claims that in order to classify data in the field of pattern recognition, such partial space methods as classifying data according to their dimensions or, in other words, according to the number of data attributes, is effective. Once data are classified in this manner, quantum logic which is logic of partial space holds between classes thus classified, and the law of partition no longer holds. Watanabe states, moreover, that the semantic duality of pattern as has been discussed in Gestaldt psychology might be described by quantum logic.

It would be worthwhile to mention that the law of partition is equivalent to the linearity law thus failure in the law of partition implies failure of the linearity law. In other words, quantum logic treats a nonlinear operator in a linear subspace. As is well known, uncertainty in chaos dynamics is the consequence of nonlinearity between state variables. Therefore, in a chaos system, it might be self-evident that the system is governed by logic in which the law of partition does not hold. From this viewpoint also, quantum logic might duly be applied to chaos.

Comparison between fundamental laws of information dynamics

Tomita [12, 13] has proposed fundamental laws of biological information processing based on analogy to thermodynamics. They consist of two fundamental laws derived from microscopic chaos (molecular chaos). His theory, called information dynamics, is derived from macroscopic chaos, and states that there exist two independent laws describing the relative relation between information quantity and the accompanying degree of freedom, governing biological information processing. In this section, compatibility of the author's theory with that of Tomita will be discussed, referring to the mission of the future biological information processing machine.

First, Tomita's theory will be briefly reviewed. The first law of information dynamics claims that when a system requires conservation of information during its transmission, the system cannot accept any limitation upon the degree of freedom in control accompanying the transmission. For example in the case of morphogenesis, a system which requires exact copy of information, must conserve accuracy of information transmission at each stage of multiply nested internal structures from individual, organ, tissue, cell, nucleus, and finally down to DNA of the lowest molecular level representing genotype. On the other hand, the second law of information dynamics claims that if the system poses any limitation upon degree of freedom, the system cannot conserve information during transmission. For example if the control is so unsophisticated as to be unable to distinguish between a cell with a nucleus and that without a nucleus, self-reproduction will never be guaranteed.

Tomita argues that information and degree of freedom are linked to each other by chaos dynamics which provides the system with a nested structure of information transmission paths. The point of his argument is that information and degree of freedom cannot be independent because of this chaos mechanism.

Let us consider the relationship between information $\Delta I(>0)$ and degree of freedom $\Delta F(>0)$ semi-quantitatively. The first law claims that degree of freedom ΔF must be greater than a certain constant when transmitted information, ΔI, is fixed to be ΔI_f. In other words it claims that such inequality as $\Delta F \cdot \Delta I_f$ or $\Delta F/\Delta I_f \geqq$ const holds.

The second law claims that transmitted information cannot exceed a certain constant when degree of freedom, ΔF, is fixed to be ΔF_f. In other words it claims that the such inequality as $\Delta I \cdot \Delta F_f$ or $\Delta I/\Delta F_f \leqq$ const holds. Among these two sets of two-fold inequalities, two inequalities such as $\Delta F/\Delta I \geqq$ const in the first law and $\Delta I/\Delta F \leqq$ const in the second law (subscript f being dropped) exclusively are the combination which holds simultaneously tautologically.

Consequently it is shown that such inequality as follows holds between information, ΔI, and degree of freedom, ΔF, K' being constant:

$$\Delta F \cdot (1/\Delta I) \geqq K'. \tag{8}$$

Degree of freedom, ΔF, in this inequality means length of a chain of nested structures in biological matter which transmits information. Moreover, the chain of nested structure is duly expressed in the recursive procedure. In general, however, a one unit progress of a recursive procedure is equivalent to a one unit progress in time as is shown by Mandelbrot [16]. Therefore, chain length of a nested structrure, or in other words depth of recursive structure is duly substituted by time Δt. As a result, inequality (8) is revised as follows, K'' being constant:

$$\Delta t \cdot (1/\Delta I) \geqq K''. \tag{9}$$

One can easily find a similarity between this inequality (9) and inequality (2) which appeared in the previous section.

This similarity implies that Tomita's fundamental laws of information dynamics are another representation of the macroscopic uncertainty principle caused by chaos dynamics. Furthermore, Tomita's theory, substituting time for degree of freedom, brings us to a novel viewpoint to look over the nature of biological information processing. That is, perfect information storage and retrieval essentially requires a long time, while information transmission within a short time does not in principle guarantee information conservation. Evidence for the former is available in the case of morphogenesis, while evidence for the latter comes from information processing in the brain.

These understandings further imply that the future biological information processing machine should have a mission of information processing completed within a finite time without conserving initial information, which is equivalent to information production as chaos dynamics indicates. On the other hand, such an information processing machine as conserves information strictly has basically already been developed as a conventional digital computer.

Relation to inductive inference

It is shown that chaos dynamics which governs biological information processing produces macroscopic uncertainty, and that the uncertainty is able to be described by quantum logic. Although quantum logic itself lies inside the framework of deductive logic, the stochastic nature of this logic implies that quantum logic may be a link between deductive logic and inductive logic. It has been demonstrated elsewhere by the present author [7] that chaos dynamics does execute inductive inference.

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